# Optimal Control of Two Area Interconnected Power System

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Abstract—This paper presents optimal and sub-optimal Automatic Generation Control (AGC) regulator schemes for a two area interconnected power system using state feedback control strategy. The power system consisting of one non-reheat thermal turbine and one hydro turbine interconnected by tie line is considered for the investigations. The formulation of optimal AGC problem involves solution of state space equations which are solved using linear quadratic regulator. The implementation of optimal AGC regulator requires observing of all the state variables of the system, which may increase the cost and complexity of the system. Due to these limitations of optimal AGC regulators, sub-optimal or incomplete state feedback AGC regulators are designed using the feedback of system states which are accessible and available for measurement. A 0.01 p.u MW load disturbance in both the areas is given and time responses are used to study the dynamic performance of the considered system. It is observed from the results that optimal control strategy gives better dynamic performance and is able to stabilize the system.

### 1. INTRODUCTION

Power systems consist of control areas representing a systematic group of alternators i.e. alternators which are characterized by equal frequency deviations due to any disturbance or fault in the system. In addition to their own generations and to eliminate mismatch between generation and demand these control areas are interconnected through tielines for providing contractual exchange of power under normal operating conditions. There are two variables of interest, namely, frequency and tie-line power exchanges [1, 2]. The purpose of AGC is to maintain system frequency very close to a specified nominal value to maintain generation of individual units' at the most economical value, to keep the correct value of the line power between different control areas. It is necessary to obtain much better frequency constancy than obtained by speed governor itself. To accomplish this, the action of the speed governor is to be supplemented by the appropriate supplementary action through modifications of speed changer settings with suitable control strategy. AGC has evolved rapidly from the time when the control function was performed manually, through the days of simple analog systems to the present application of sophisticated direct digital control techniques. Many papers have been published

on the investigations in the area of AGC problem of interconnected power systems. Over the past decades, many control strategies have been proposed for AGC based on classical control viz. Integral (I), Proportional Integral and Derivative (PID) [3, 4]. These controllers give slow and sluggish response. Due to these limitations, more advanced controllers are being investigated to give a higher degree of accuracy and faster response to maintain the frequency and tie-line power at their nominal values.

Based on the modern control strategies, optimal and suboptimal or incomplete state feedback control are used to eliminate the error in frequency and tie line power caused by any random disturbance in the power system [5, 6]. The optimal AGC regulator designs are based on linear quadratic regulator theory which requires the feedback of either all the states variables of the system which may increase the cost and complexity of the system. Due to these limitations of optimal AGC regulators, sub-optimal or incomplete state feedback AGC regulators are designed using the feedback of system states which are accessible and available for measurement.

The objective of this paper is to present AGC regulation design techniques based on modern control. The optimal and sub-optimal control schemes are studied for maintaining the frequency deviation and tie-line power exchanges within nominal range.

The paper is organized as follows. Firstly, the plant system model consisting of hydrothermal power system is explained. Further, simulation results are presented followed by conclusions.

#### 2. POWER SYSTEM UNDER INVESTIGATION

A two area interconnected power system consisting of thermal and a hydro turbine is considered. The power system areas are interconnected via tie line. The incremental power deviation in tie line is modeled with the difference of frequency deviations in both power system areas given by Eq. (1).

$$\Delta P_{tie} = \frac{2 * \pi * T_0}{s} \left( \Delta f_1 - \Delta f_2 \right) \tag{1}$$

where  $\Delta P_{tie}$  is the tie line power flow between area 1 and area 2 and  $T_0$  is the synchronizing coefficient for tie line for two area system. Conventional AGC is based upon tie-line bias control, which each area tends to reduce the Area Control Error (ACE) to zero [7]. The control error for each area consists of a linear combination of frequency and tie-line error given by Eq. (2).

$$ACE_i = \Delta P_{tie} + B\Delta f_i \tag{2}$$

where B is the tie line frequency bias in area 1 and area 2. The transfer function model of the interconnected power system is shown in Fig. 1. The parameters are given in appendix. The modeling of thermal and hydro turbine is based on the models given by [8].

#### 2.1 State space modeling of two area hydrothermal system

The state space model of two area hydro-thermal power system with full state and variable structured feedback has been developed in this section.

**Optimal control:** The linear model of the system is described in the state space form, as

$$\dot{x} = Ax + Bu + \Gamma d \tag{3}$$

where, A  $(n \times n)$  is state matrix, B  $(n \times m)$  is control matrix for n number of state variables and m number of inputs and  $\Gamma$ is disturbance matrix. Different variables have been defined as:

State variables:

$$\begin{aligned} x_1 &= \Delta f_1 \ x_2 &= \Delta P_{t1} \ x_3 &= \Delta P_{g1} \ x_4 &= \Delta f_2 \ x_5 &= \Delta P_{tw} \ x_6 &= \Delta P_{g2} \\ x_7 &= \Delta P_{G1} \ x_8 &= \Delta P_{tie} \ x_9 &= \int ACE_1 dt \ x_{10} &= \int ACE_2 dt \end{aligned}$$

Control inputs are:  $u_1$  and  $u_2$ 

Disturbance inputs:  $d_1$  and  $d_2$ 

The system state space equations with reference to transfer function blocks in Fig. 2 are given below:

$$x_{1} = \frac{K_{p1}}{1 + sT_{p1}} (x_{2} - x_{8} - d_{1})$$

$$x_{1} + T_{p1}\dot{x}_{1} = K_{p1} (x_{2} - x_{8} - d_{1})$$

$$\dot{x}_{1} = -\frac{1}{T_{p1}} x_{1} + \frac{K_{p1}}{T_{p1}} x_{2} - \frac{K_{p1}}{T_{p1}} x_{8} - \frac{K_{p1}}{T_{p1}} d_{1}$$
(4)

In a similar way, state equations for all the variables can be written as:

$$\dot{x}_2 = -\frac{1}{T_{t1}} x_2 + \frac{1}{T_{t1}} x_3 \tag{5}$$

$$\dot{x}_3 = -\frac{1}{T_{g_1}R_1} x_1 - \frac{1}{T_{g_1}} x_3 + \frac{1}{T_{g_1}} u_1$$
(6)

$$x_{4} + T_{p2}\dot{x}_{4} = K_{p2}\left(-x_{5} + x_{8} - d_{2}\right)$$
  
$$\dot{x}_{4} = -\frac{1}{T_{r2}}x_{4} + \frac{K_{p2}}{T_{r2}}x_{5} + \frac{K_{p2}}{T_{r2}}x_{8} - \frac{K_{p2}}{T_{r2}}d_{2}$$
(7)

$$\dot{x}_{5} = \frac{2T_{2}}{T_{1}T_{3}R_{2}}x_{4} - \frac{2}{T_{w}}x_{5} + \left(\frac{2}{T_{w}} + \frac{2}{T_{3}}\right)x_{6} + \left(\frac{2T_{2}}{T_{1}T_{3}} - \frac{2}{T_{3}}\right)x_{7} - \frac{2T_{2}}{T_{1}T_{3}}u_{2}$$
(8)

$$\dot{x}_6 = -\frac{T_2}{T_1 T_3 R_2} x_4 - \frac{1}{T_3} x_6 + \left(\frac{1}{T_3} - \frac{T_2}{T_1 T_3}\right) x_7 + \frac{T_2}{T_1 T_3} u_2$$
(9)

$$\dot{x}_7 = -\frac{1}{T_1 R_2} x_4 - \frac{1}{T_1} x_7 + \frac{1}{T_1} u_2 \tag{10}$$

$$\dot{x}_8 = 2\pi T_0 x_1 - 2\pi T_0 x_4 \tag{11}$$

$$\dot{x}_9 = B_1 x_1 + x_8 \tag{12}$$

$$\dot{x}_{10} = B_2 x_4 - x_8 \tag{13}$$

Using the Eq. (4) to Eq. (13), the matrices A, B and  $\Gamma$  are constructed. The system is analytically studied using state matrix which determines the stability of the system.

The optimal control is given by u=-Kx where 'K' is the feedback gain matrix given by  $K = R^{-1}B^T S$  where, 'P' is a real, symmetric and positive definite matrix which is the unique solution of matrix Riccati equation given as follows:

$$A^T P + PA - PBR^{-1}B^T P + Q = 0$$
<sup>(14)</sup>

The closed loop system equation is:

$$\dot{x} = Ax + B(-Kx) = (A - BK)x = A_c x$$
 (15)

The matrix  $A_c$  is the closed loop system matrix whose eigen values gives a test for the stability of the closed loop system. The more negative magnitude of real part of eigen value indicates the more stable system. The problem of designing an optimal controller can be defined as to determine the feedback matrix 'K' so as to move the system from an initial state to desired state while satisfying the performance index (PI) and meeting out certain control and state constraints [9]. Generally the PI is of quadratic form as:

$$PI = \frac{1}{2} \int \left( x^T Q x + u^T R u \right) dt \tag{16}$$

where, 'Q' is a real, symmetric and positive semi-definite matrix called as state weighting matrix and 'R' is a real, symmetric and positive definite matrix called as control weighting matrix. These matrices are determined on the basis of requirement that the deviations of ACEs,  $\int ACE \, dt$  and control inputs about steady values are minimized. Thus, PI takes the following form:



Fig. 1: Two area power system with optimal control

$$PI = \frac{1}{2} \int_{0}^{\infty} [(B_1 x_1 + x_8)^2 + (B_2 x_4 - x_8)^2 + x_9^2 + x_{10}^2 + u_1^2 + u_2^2] dt \quad (17)$$

**Sub-optimal control:** The implementation of optimal AGC regulator requires monitoring of all the state variables of the system or state reconstruction, which may be undesirable from cost and complexity considerations. The sub-optimal or incomplete state feedback control considers only the crucial system states. These controllers are preferable in practice since they require information of observable states only [10]. For this variable structured control, the state variables considered are:

$$x_1 = \Delta f_1 \quad x_2 = \Delta f_2 \quad x_3 = \Delta P_{iie1} \quad x_4 = \int ACE_1 dt \quad x_5 = \int ACE_2 dt$$

The frequency and tie line power deviations are to be minimized and which are in turn controlled by the ACEs. So, the state variables for corresponding desired states are considered. Hence the state equations for incomplete state feedback are:

$$\dot{x}_{1} = -\frac{1}{T_{p1}} x_{1} - \frac{K_{p1}}{T_{p1}} x_{3} - \frac{K_{p1}}{T_{p1}} d_{1}$$
(18)

$$\dot{x}_2 = -\frac{1}{T_{p2}}x_2 + \frac{K_{p2}}{T_{p2}}x_3 - \frac{K_{p2}}{T_{p2}}d_2$$
(19)

$$\dot{x}_3 = 2\pi T_0 x_1 - 2\pi T_0 x_2 \tag{20}$$

Then similar to optimal strategy, riccati equation is solved to obtain the closed loop matrix which gives the eigen values for the system.

#### **3. SIMULATION RESULTS**

 $\dot{x}_4 = B_1 x_1 + x_3$ 

 $\dot{x}_5 = B_2 x_2 - x_3$ 

The two area hydrothermal interconnected power system is used to investigate the effect of optimal and sub-optimal control on frequency and tie line power deviation when there is 1% change in load in both the areas. The parameters for thermal and hydro turbine are given in the Appendix. The optimal and sub-optimal feedback gains for the considered system are given in Table 1. For analysing the closed loop stability, the closed loop eigen values are also obtained given in Table 2.

Fig. 2 to Fig. 4 gives the response of  $\Delta f_1$ ,  $\Delta f_2$  and  $\Delta P_{tie}$  for a load perturbation of 1% in area 1. In a similar way, responses for load perturbation in area 2 can also be plotted. It can be observed that the oscillations are suppressed by optimal control in around 18 seconds whereas the sub-optimal controller takes more time for steady state achievement. The peak overshoot is also more for  $\Delta f_1$  and  $\Delta f_2$  deviation by the sub-optimal control technique whereas the tie-line power is effectively controlled using the incomplete state feedback control (Fig. 4).

(21)

(22)

					0					
Controller	Feedback gain matrix									
Optimal	0.6597	1.1572	0.2095	0.1842	0.9205	1.9204	-0.5813	0.9674	0.7877	0.6160
	-0.0052	-0.0059	-0.0009	0.2240	0.6794	11.7167	8.6338	0.5555	-0.6160	0.7877
Sub-optimal	0.8505	0.1156	-0.2366	1.0000	-0.0000					
	0.1156	0.8505	0.2366	-0.0000	1.0000					

 Table 1: Feedback gain matrices

Table 2: Closed loop eigen values

Optimal controller	Sub-optimal controller
-13.0676	$-0.3109 \pm 2.3019i$
$-1.0386 \pm 3.1050i$	$-0.5081 \pm 0.4085i$
$-0.9071 \pm 1.4148i$	-0.1631
-2.0001	
-0.7583	
$-0.0778 \pm 0.1130i$	
-0.1545	



Fig. 2: Response of frequency deviation of area 1 for 1% load disturbance in area 1







Fig. 4: Response of tie line power deviation for 1% load disturbance in area 1

#### 4. RESULTS AND DISCUSSION

The feedback gains, as shown in Table 1, obtained for optimal and suboptimal AGC regulator schemes shows that there is an appreciable reduction in the values of feedback gains of suboptimal AGC regulators. The closed loop eigen values Table 2, reveals that stability margins are appreciably reduced in case of constrained feedback control strategy as compared to that obtained in case of full state vector feedback control strategy. The noticeable reduction in magnitudes of negative real parts and increased magnitudes of imaginary parts of eigenvalues with constrained feedback control strategy as compared to the magnitudes of corresponding parts obtained in case of optimal control scheme based on complete state feedback control strategy, led to the deterioration in steady state system dynamic performance. These investigations reveal that the overall degree of stability of the system with sub-optimal AGC regulators using constrained feedback control strategy is decreased to a great extent. The time responses also depict that the responses of optimal and suboptimal control stabilize the system.

#### 5. ACKNOWLEGEMENT

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## Appendix:

Parameter	Description	Value	Unit
R <sub>1</sub> ,R <sub>2</sub>	Regulations of governors in areas 1,2	2.4	Hz/pu MW
$B_{1}, B_{2}$	Tie line frequency bias in areas 1,2	0.425	pu MW/Hz
T <sub>g1</sub>	Governor time constants for thermal areas 1	0.08	second
T <sub>t1</sub>	Turbine time constants for thermal areas 1	0.4	second
$T_1$	Hydro governor (Stage 1) time constant for area 2	48.7	second
$T_2$	Hydro governor (Stage 2) time constant for area 2	0.513	second
<b>T</b> <sub>3</sub>	Hydro governor (Stage 2) time constant for area 2	10	second
T <sub>w</sub>	Water starting time for water turbine in area 2	1	second
T <sub>0</sub>	Synchronizing coefficients for tie lines	0.0707	MW/radian